CHAPTER 4

SHALLOW FOUNDATIONS

4-1. **Basic Considerations.** Shallow foundations may consist of spread footings supporting isolated columns, combined footings for supporting loads from several columns, strip footings for supporting walls, and mats for supporting the entire structure.

a. **Significance and Use.** These foundations may be used where there is a suitable bearing stratum near the ground surface and settlement from compression or consolidation of underlying soil is acceptable. Potential heave of expansive foundation soils should also be acceptable. Deep foundations should be considered if a suitable shallow bearing stratum is not present or if the shallow bearing stratum is underlain by weak, compressible soil.

b. **Settlement Limitations.** Settlement limitation requirements in most cases control the pressure which can be applied to the soil by the footing. Acceptable limits for total downward settlement or heave are often 1 to 2 inches or less. Refer to EM 1110-1-1904 for evaluation of settlement or heave.

(1) **Total Settlement.** Total settlement should be limited to avoid damage with connections in structures to outside utilities, to maintain adequate drainage and serviceability, and to maintain adequate freeboard of embankments. A typical allowable settlement for structures is 1 inch.

(2) **Differential Settlement.** Differential settlement nearly always occurs with total settlement and must be limited to avoid cracking and other damage in structures. A typical allowable differential/span length ratio Δ/L for steel and concrete frame structures is 1/500 where Δ is the differential movement within span length L.

c. **Bearing Capacity.** The ultimate bearing capacity should be evaluated using results from a detailed in situ and laboratory study with suitable theoretical analyses given in 4-2. Design and allowable bearing capacities are subsequently determined according to Table 1-1.

4-2. **Solution of Bearing Capacity.** Shallow foundations such as footings or mats may undergo either a general or local shear failure. Local shear occurs in loose sands which undergo large strains without complete failure. Local shear may also occur for foundations in sensitive soils with high ratios of peak to residual strength. The failure pattern for general shear is modeled by Figure 1-3. Solutions of the general equation are provided using the Terzaghi, Meyerhof, Hansen and Vesic models. Each of these models have different capabilities for considering foundation geometry and soil conditions. Two or more models should be used for each design case when practical to increase confidence in the bearing capacity analyses.

a. **General Equation.** The ultimate bearing capacity of the foundation shown in Figure 1-6 can be determined using the general bearing capacity Equation 1-1

$$q_u = cN_c\zeta_c + \frac{1}{2}B'\gamma'_H N_\gamma \zeta_\gamma + \sigma'_D N_q \zeta_q \qquad (4\text{-}1)$$

where

- q_u = ultimate bearing capacity, ksf
- c = unit soil cohesion, ksf
- B' = minimum effective width of foundation $B - 2e_B$, ft
- e_B = eccentricity parallel with foundation width B, M_B/Q, ft
- M_B = bending moment parallel with width B, kips-ft
- Q = vertical load applied on foundation, kips
- γ'_H = effective unit weight beneath foundation base within the failure zone, kips/ft^3
- σ'_D = effective soil or surcharge pressure at the foundation depth D, $\gamma'_D \cdot D$, ksf
- γ'_D = effective unit weight of soil from ground surface to foundation depth, kips/ft^3
- D = foundation depth, ft
- N_c, N_γ, N_q = dimensionless bearing capacity factors of cohesion c, soil weight in the failure wedge, and surcharge q terms
- $\zeta_c, \zeta_\gamma, \zeta_q$ = dimensionless correction factors of cohesion c, soil weight in the failure wedge, and surcharge q accounting for foundation geometry and soil type

(1) **Net Bearing Capacity.** The net ultimate bearing capacity q'_u is the maximum pressure that may be applied to the base of the foundation without undergoing a shear failure that is in addition to the overburden pressure at depth D.

$$q'_u = q_u - \gamma_D \cdot D \qquad (4\text{-}2)$$

(2) **Bearing Capacity Factors.** The dimensionless bearing capacity factors N_c, N_q, and N_γ are functions of the effective friction angle ϕ' and depend on the model selected for solution of Equation 4-1.

(3) **Correction Factors.** The dimensionless correction factors ζ consider a variety of options for modeling actual soil and foundation conditions and depend on the model selected for solution of the ultimate bearing capacity. These options are foundation shape with eccentricity, inclined loading, foundation depth, foundation base on a slope, and a tilted foundation base.

b. **Terzaghi Model.** An early approximate solution to bearing capacity was defined as general shear failure (Terzaghi 1943). The Terzaghi model is applicable to level strip footings placed on or near a level ground surface where foundation depth D is less than the minimum width B. Assumptions include use of a surface footing on soil at plastic equilibrium and a failure surface similar to Figure 1-3a. Shear resistance of soil above the base of an embedded foundation is not included in the solution.

(1) **Bearing Capacity Factors.** The Terzaghi bearing capacity factors N_c and N_q for general shear are shown in Table 4-1 and may be calculated by

TABLE 4-1

Terzaghi Dimensionless Bearing Capacity Factors (after Bowles 1988)

ϕ'	N_q	N_c	N_γ	ϕ'	N_q	N_c	N_γ
28	17.81	31.61	15.7	0	1.00	5.70	0.0
30	22.46	37.16	19.7	2	1.22	6.30	0.2
32	28.52	44.04	27.9	4	1.49	6.97	0.4
34	36.50	52.64	36.0	6	1.81	7.73	0.6
35	41.44	57.75	42.4	8	2.21	8.60	0.9
36	47.16	63.53	52.0	10	2.69	9.60	1.2
38	61.55	77.50	80.0	12	3.29	10.76	1.7
40	81.27	95.66	100.4	14	4.02	12.11	2.3
42	108.75	119.67	180.0	16	4.92	13.68	3.0
44	147.74	151.95	257.0	18	6.04	15.52	3.9
45	173.29	172.29	297.5	20	7.44	17.69	4.9
46	204.19	196.22	420.0	22	9.19	20.27	5.8
48	287.85	258.29	780.1	24	11.40	23.36	7.8
50	415.15	347.51	1153.2	26	14.21	27.09	11.7

$$N_c = (N_q - 1)\cot\phi' \tag{4-3a}$$

$$N_q = \frac{e^{\frac{270-\phi'}{180}\pi\tan\phi'}}{2\cos^2(45+\phi'/2)} \tag{4-3b}$$

Factor N_γ depends largely on the assumption of the angle ψ in Figure 1-3a. N_γ varies from minimum values using Hansen's solution to maximum values using the original Terzaghi solution. N_γ shown in Table 4-1, was backfigured from the original Terzaghi values assuming $\psi = \phi'$ (Bowles 1988).

(2) **Correction Factors.** The Terzaghi correction factors ζ_c and ζ_γ consider foundation shape only and are given in Table 4-2. $\zeta_q = 1.0$ (Bowles 1988).

TABLE 4-2

Terzaghi Correction Factors ζ_c and ζ_γ

Factor	Strip	Square	Circular
ζ_c	1.0	1.3	1.3
ζ_γ	1.0	0.8	0.6

c. **Meyerhof Model.** This solution considers correction factors for eccentricity, load inclination, and foundation depth. The influence of the shear strength of soil above the base of the foundation is considered in this solution. Therefore, beneficial effects of the foundation depth can be included in the analysis. Assumptions include use of a shape factor ζ_q for surcharge, soil at plastic equilibrium, and a log spiral failure surface that includes shear above the base of the foundation. The angle $\psi = 45 + \phi/2$ was used for determination of N_γ. Table 4-3 illustrates the Meyerhof dimensionless bearing capacity and correction factors required for solution of Equation 4-1 (Meyerhof 1963).

(1) **Bearing Capacity Factors.** Table 4-4 provides the bearing capacity factors in 2-degree intervals.

(2) **Correction Factors.** Correction factors are given by

Cohesion: $\zeta_c = \zeta_{cs} \cdot \zeta_{ci} \cdot \zeta_{cd}$
Wedge: $\zeta_\gamma = \zeta_{\gamma s} \cdot \zeta_{\gamma i} \cdot \zeta_{\gamma d}$
Surcharge: $\zeta_q = \zeta_{qs} \cdot \zeta_{qi} \cdot \zeta_{qd}$

where subscript s indicates shape with eccentricity, subscript i indicates inclined loading, and d indicates foundation depth.

(3) **Eccentricity.** The influence of bending moments on bearing capacity can be estimated by converting bending moments to an equivalent eccentricity e. Footing dimensions are then reduced to consider the adverse effect of eccentricity.

(a) Effective footing dimensions may be given by

$$B' = B - 2e_B \quad (4\text{-}4a)$$

$$W' = W - 2e_W \quad (4\text{-}4b)$$

$$e_B = \frac{M_B}{Q} \quad (4\text{-}4c)$$

$$e_W = \frac{M_W}{Q} \quad (4\text{-}4d)$$

where

M_B = bending moment parallel with foundation width B, kips-ft
M_W = bending moment parallel with foundation length W, kips-ft

Orientation of axes, eccentricities,and bending moments are shown in Table 4-3.

(b) The ultimate load applied to footings to cause a bearing failure is

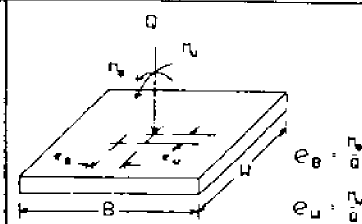
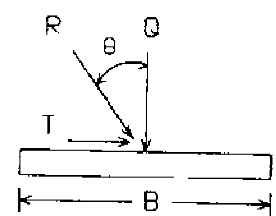
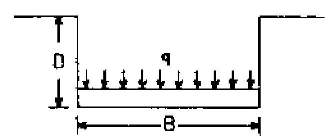
$$Q_u = q_u \cdot A_e \quad (4\text{-}5)$$

where

q_u = ultimate bearing capacity of Equation 4-1 considering eccentricity in the foundation shape correction factor, Table 4-3, ksf
A_e = effective area of the foundation $B'W'$, ft^2

TABLE 4-3

Meyerhof Dimensionless Bearing Capacity and Correction Factors (Data from Meyerhof 1953; Meyerhof 1963)

FACTOR			COHESION (c)	WEDGE (γ)	SURCHARGE (q)	DIAGRAM
BEARING CAPACITY N			N_c	N_γ	N_q	
		$\phi = 0$	5.14	0.00	1.00	
		$\phi > 0$	$(N_q-1)\cot\phi$	$(N_q-1)\tan(1.4\phi)$	$N_\phi e^{\pi\tan\phi}$	
CORRECTION ζ	FOUNDATION SHAPE WITH ECCENTRICITY s		ζ_{cs}	$\zeta_{\gamma s}$	ζ_{qs}	
		$\phi = 0$	$1 + 0.2N_\phi \frac{B'}{W'}$	1.0	1.0	
		$\phi > 10$	"	$1 + 0.1N_\phi \frac{B'}{W'}$	$1 + 0.1N_\phi \frac{B'}{W'}$	
		$0 < \phi \le 10$	"	Linear Interpolation Between $\phi = 0$ and $\phi = 10$ Degrees		
	INCLINED LOADING i		ζ_{ci}	$\zeta_{\gamma i}$	ζ_{qi}	
		$\phi = 0$	$\left[1 - \frac{\theta}{90}\right]$	1.0	$\left[1 - \frac{\theta}{90}\right]$	
		$\phi > 0$	$\left[1 - \frac{\theta}{90}\right]^2$	$\theta \le \phi$ $\left[1 - \frac{\theta}{\phi}\right]^2$ $\theta > \phi$ 0.0	$\left[1 - \frac{\theta}{90}\right]^2$	
	FOUNDATION DEPTH d		ζ_{cd}	$\zeta_{\gamma d}$	ζ_{qd}	
		$\phi = 0$	$1 + 0.2(N_\phi)^{1/2} \cdot \frac{D}{B}$	1.0	1.0	
		$\phi > 0$	"	$1 + 0.1(N_\phi)^{1/2} \cdot \frac{D}{B}$	$1 + 0.1(N_\phi)^{1/2} \cdot \frac{D}{B}$	
		$0 < \phi \le 10$	"	Linear Interpolation Between $\phi = 0$ and $\phi = 10$ Degrees		

Note: Eccentricity and inclined loading correction factors may not be used simultaneously; factors not used are unity

Nomenclature:

- ϕ = angle of internal friction, degrees
- N_ϕ = $\tan^2(45 + \phi/2)$
- B' = effective width of foundation, $B - 2e_B$, ft
- W' = effective lateral length of foundation, $W - 2e_W$, ft
- B = foundation width, ft
- W = foundation lateral length, ft
- D = foundation depth, ft
- Q = vertical load on foundation, qBW, kips
- q = bearing pressure on foundations, ksf
- T = horizontal load on foundation, right ↕, kips
- R = resultant load on foundation, $(Q^2 + T^2)^{1/2}$
- θ = angle of resultant load with vertical axis, $\cos^{-1}(Q/R)$, degrees
- e_B = eccentricity parallel with B, M_B/Q
- e_W = eccentricity parallel with W, M_W/Q
- M_B = bending moment parallel with B, kips-ft
- M_W = bending moment parallel with W, kips-ft

TABLE 4-4

Meyerhof, Hansen, and Vesic Dimensionless Bearing Capacity Factors

				N_γ		
ϕ	N_ϕ	N_c	N_q	Meyerhof	Hansen	Vesic
0	1.00	5.14	1.00	0.00	0.00	0.00
2	1.07	5.63	1.20	0.01	0.01	0.15
4	1.15	6.18	1.43	0.04	0.05	0.34
6	1.23	6.81	1.72	0.11	0.11	0.57
8	1.32	7.53	2.06	0.21	0.22	0.86
10	1.42	8.34	2.47	0.37	0.39	1.22
12	1.52	9.28	2.97	0.60	0.63	1.69
14	1.64	10.37	3.59	0.92	0.97	2.29
16	1.76	11.63	4.34	1.37	1.43	3.06
18	1.89	13.10	5.26	2.00	2.08	4.07
20	2.04	14.83	6.40	2.87	2.95	5.39
22	2.20	16.88	7.82	4.07	4.13	7.13
24	2.37	19.32	9.60	5.72	5.75	9.44
26	2.56	22.25	11.85	8.00	7.94	12.54
28	2.77	25.80	14.72	11.19	10.94	16.72
30	3.00	30.14	18.40	15.67	15.07	22.40
32	3.25	35.49	23.18	22.02	20.79	30.21
34	3.54	42.16	29.44	31.15	28.77	41.06
36	3.85	50.59	37.75	44.43	40.05	56.31
38	4.20	61.35	48.93	64.07	56.17	78.02
40	4.60	75.31	64.19	93.69	79.54	109.41
42	5.04	93.71	85.37	139.32	113.95	155.54
44	5.55	118.37	115.31	211.41	165.58	224.63
46	6.13	152.10	158.50	328.73	244.64	330.33
48	6.79	199.26	222.30	526.44	368.88	495.99
50	7.55	266.88	319.05	873.84	568.56	762.85

(c) The bearing capacity of eccentric loaded foundations may also be estimated by (Meyerhof 1953)

$$q_{ue} = q_u \cdot R_e \tag{4-6}$$

where R_e is defined for cohesive soil by

$$R_e = 1 - 2 \cdot \frac{e}{B} \tag{4-7a}$$

and for cohesionless soil (0 < e/B < 0.3) by

$$R_e = 1 - \sqrt{\frac{e}{B}} \tag{4-7b}$$

where

q_u = ultimate capacity of a centrally loaded foundation found from Equation 4-1 ignoring bending moments, ksf
e = eccentricity from Equations 4-4c and 4-4d, ft

d. **Hansen Model.** The Hansen model considers tilted bases and slopes in addition to foundation shape and eccentricity, load inclination, and foundation depth. Assumptions are based on an extension of Meyerhof's work to include tilting of the base and construction on a slope. Any D/B ratio may be used permitting bearing capacity analysis of both shallow and deep foundations. Bearing capacity factors N_c and N_q are the same as Meyerhof's factors. N_γ is calculated assuming $\psi = 45 + \phi/2$. These values of N_γ are least of the methods. Correction factors ζ_c, ζ_γ, and ζ_q in Equation 4-1 are

$$\text{Cohesion: } \zeta_c = \zeta_{cs} \cdot \zeta_{ci} \cdot \zeta_{cd} \cdot \zeta_{c\beta} \cdot \zeta_{c\delta} \tag{4-8a}$$

$$\text{Wedge: } \zeta_\gamma = \zeta_{\gamma s} \cdot \zeta_{\gamma i} \cdot \zeta_{\gamma d} \cdot \zeta_{\gamma\beta} \cdot \zeta_{\gamma\delta} \tag{4-8b}$$

$$\text{Surcharge: } \zeta_q = \zeta_{qs} \cdot \zeta_{qi} \cdot \zeta_{qd} \cdot \zeta_{q\beta} \cdot \zeta_{q\delta} \tag{4-8c}$$

where subscripts s, i, d, β, and δ indicate shape with eccentricity, inclined loading, foundation depth, ground slope, and base tilt, respectively. Table 4-5 illustrates the Hansen dimensionless bearing capacity and correction factors for solution of Equation 4-1.

(1) **Restrictions.** Foundation shape with eccentricity ζ_{cs}, $\zeta_{\gamma s}$, and ζ_{qs} and inclined loading ζ_{ci}, $\zeta_{\gamma i}$, and ζ_{qi} correction factors may not be used simultaneously. Correction factors not used are unity.

(2) **Eccentricity.** Influence of bending moments is evaluated as in the Meyerhof model.

(3) **Inclined loads.** The B component in Equation 4-1 should be width B if horizontal load T is parallel with B or should be W if T is parallel with length W.

e. **Vesic Model.** Table 4-6 illustrates the Vesic dimensionless bearing capacity and correction factors for solution of Equation 4-1.

(1) **Bearing Capacity Factors.** N_c and N_q are identical with Meyerhof's and Hansen's factors. N_γ was taken from an analysis by Caquot and Kerisel (1953) using $\psi = 45 + \phi/2$.

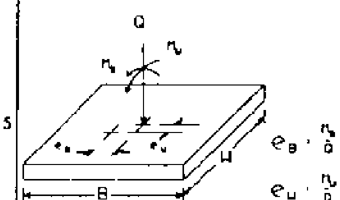
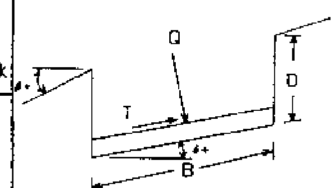
(2) **Local Shear.** A conservative estimate of N_q may be given by

$$N_q = (1 + \tan\phi') \cdot e^{\tan\phi'} \cdot \tan^2\left[45 + \frac{\phi'}{2}\right] \tag{4-9}$$

Equation 4-9 assumes a local shear failure and leads to a lower bound estimate of q_u. N_q from Equation 4-9 may also be used to calculate N_c and N_γ by the equations given in Table 4-6.

TABLE 4-5

Hansen Dimensionless Bearing Capacity and Correction Factors (Data from Hansen 1970)

FACTOR			COHESION (c)	WEDGE (γ)	SURCHARGE (q)	DIAGRAM
BEARING CAPACITY N			N_c	N_γ	N_q	
		$\phi = 0$	5.14	0.00	1.00	
		$\phi > 0$	$(N_q-1)\cot\phi$	$1.5(N_q-1)\tan\phi$	$N_\phi e^{\pi\tan\phi}$	
CORRECTION ζ	FOUNDATION SHAPE WITH ECCENTRICITY s		ζ_{cs}	$\zeta_{\gamma s}$	ζ_{qs}	
			Strip: 1.0			
		$\phi = 0$	$0.2\cdot\frac{B'}{W'}$	1.0	1.0	
		$\phi > 0$	$1 + \frac{N_q B'}{N_c W'}$	$1 - 0.4\cdot\frac{B'}{W'}$	$1 + \frac{B'}{W'}\tan\phi$	
	INCLINED LOADING i		ζ_{ci}	$\zeta_{\gamma i}$	ζ_{qi}	
		$\phi = 0$	$1 - \frac{\left[1 - \frac{T}{A_e c_a}\right]^{\frac{1}{2}}}{2}$	$\delta=0$ $\left[1 - \frac{0.7T}{Q+A_e c_a\cot\phi}\right]^5$	$\left[1 - \frac{0.5T}{Q+A_e c_a\cot\phi}\right]^5$	
		$\phi > 0$	$\zeta_{qi} - \frac{1-\zeta_{qi}}{N_q - 1}$	$\delta>0$ $\left[1 - \frac{(0.7-\delta/450)T}{Q+A_e c_a\cot\phi}\right]^5$		
	FOUNDATION DEPTH d		ζ_{cd}	$\zeta_{\gamma d}$	ζ_{qd}	
		$\phi = 0$	$0.4k$	1.0	1.0	
		$\phi > 0$	$1 + 0.4k$	1.0	$1 + 2\tan\phi(1-\sin\phi)^2k$	
	BASE ON SLOPE β		$\zeta_{c\beta}$	$\zeta_{\gamma\beta}$	$\zeta_{q\beta}$	
		$\phi = 0$	$1 - \frac{\beta}{147.3}$	$(1 - 0.5\tan\beta)^5$	$(1 - 0.5\tan\beta)^5$	$\beta + \delta \le 90°$
		$\phi > 0$	$\zeta_{q\beta} - \frac{1-\zeta_{q\beta}}{147.3}$			$\beta < \phi$
	TILTED BASE δ		$\zeta_{c\delta}$	$\zeta_{\gamma\delta}$	$\zeta_{q\delta}$	
		$\phi = 0$	$1 - \frac{\delta}{147}$	$e^{-0.047\delta\tan\phi}$	$e^{-0.035\delta\tan\phi}$	
		$\phi > 0$	$\zeta_{q\delta} - \frac{1-\zeta_{q\delta}}{147.3}$			

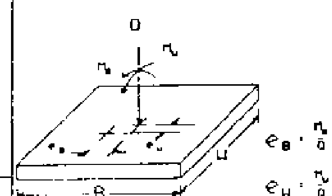
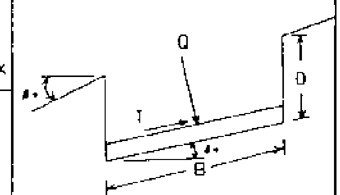
Note: Eccentricity and inclined loading correction factors may not be used simultaneously; factors not used are unity

Nomenclature:

- ϕ = angle of internal friction, degrees
- N_ϕ = $\tan^2(45 + \phi/2)$
- B' = effective width of foundation, $B - 2e_B$, ft
- W' = effective length of foundation, $W - 2e_W$, ft
- B = foundation width, ft
- W = foundation length, ft
- e_B = eccentricity parallel with B, M_B/Q
- e_W = eccentricity parallel with W, M_W/Q
- M_B = bending moment parallel with B, kips-ft
- M_W = bending moment parallel with W, kips-ft
- ϕ_a = friction angle between base and soil $\approx \phi$, degrees
- c_a = adhesion of soil to base $\le$ c, ksf
- c = soil cohesion or undrained shear strength C_u, ksf
- δ = base tilt from horizontal, upward +, degrees
- β = slope of ground from base, downward +, degrees
- k = D/B if $D/B \le 1$ OR $\tan^{-1}(D/B)$ if $D/B > 1$ (in radians)
- D = foundation depth, ft
- Q = vertical load on foundation, kips
- T = horizontal load $\le Q\tan\phi + A_e c_a$, right +, kips
- A_e = effective area of foundation $B'W'$, ft^2

TABLE 4-6

Vesic Dimensionless Bearing Capacity and Correction Factors (Data from Vesic 1973; Vesic 1975)

FACTOR			COHESION (c)	WEDGE (γ)	SURCHARGE (q)	DIAGRAM
BEARING CAPACITY N			N_c	N_γ	N_q	
		$\phi = 0$	5.14	0.00 OR $-2\sin\beta$ if $\beta>0$	1.00	
		$\phi > 0$	$(N_q-1)\cot\phi$	$2(N_q+1)\tan\phi$	$N_\phi e^{\pi \tan\phi}$	
CORRECTION ζ	FOUNDATION SHAPE WITH ECCENTRICITY s		ζ_{cs}	$\zeta_{\gamma s}$	ζ_{qs}	
		$\phi = 0$	Strip: 1.0 $0.2 \cdot \frac{B'}{W'}$	1.0	1.0	
		$\phi > 0$	$1 + \frac{N_q B'}{N_c W'}$	$1 - 0.4 \cdot \frac{B'}{W'}$ (1.0 if strip)	$1 + \frac{B'}{W'}\tan\phi$ (1.0 if strip)	
	INCLINED LOADING i		ζ_{ci}	$\zeta_{\gamma i}$	ζ_{qi}	
		$\phi = 0$	$\frac{1 - \frac{mT}{A_e c_a N_c}}{2}$	$\left[1 - \frac{T}{Q + A_e c_a \cot\phi}\right]^{m+1} > 0$	$\left[1 - \frac{T}{Q + A_e c_a \cot\phi}\right]^{m}$	
		$\phi > 0$	$\zeta_{qi} - \frac{1 - \zeta_{qi}}{N_q - 1}$			
	FOUNDATION DEPTH d		ζ_{cd}	$\zeta_{\gamma d}$	ζ_{qd}	
		$\phi = 0$	1 + 0.4K	1.0	1.0	
		$\phi > 0$	1 + 0.4k	1.0	$1 + 2\tan\phi(1-\sin\phi)^2 k$	
	BASE ON SLOPE β		$\zeta_{c\beta}$	$\zeta_{\gamma\beta}$	$\zeta_{q\beta}$	$\beta + \delta \le 90°$ $\beta < \phi$
		$\phi = 0$	$1 - \frac{\beta}{147.3}$	$(1 - \tan\beta)^2$	$(1 - \tan\beta)^2$	
		$\phi > 0$	$\zeta_{q\beta} - \frac{1 - \zeta_{q\beta}}{147.3}$			
	TILTED BASE δ		$\zeta_{c\delta}$	$\zeta_{\gamma\delta}$	$\zeta_{q\delta}$	
		$\phi = 0$	$1 - \frac{\delta}{147}$	$(1 - 0.017\delta\tan\phi)^2$	$(1 - 0.017\delta\tan\phi)^2$	
		$\phi > 0$	$\zeta_{q\delta} - \frac{1 - \zeta_{q\delta}}{147.3}$			

Note: Eccentricity and inclined loading correction factors may not be used simultaneously; factors not used are unity

Nomenclature:

- ϕ = angle of internal friction, degrees
- N_ϕ = $\tan^2(45 + \phi/2)$
- B' = effective width of foundation, $B - 2e_B$, ft
- W' = effective length of foundation, $W - 2e_W$, ft
- B = foundation width, ft
- W = foundation length, ft
- e_B = eccentricity parallel with B, M_B/Q
- e_W = eccentricity parallel with W, M_W/Q
- M_B = bending moment parallel with B, kips-ft
- M_W = bending moment parallel with W, kips-ft
- ϕ_a = friction angle between base and soil ≈ ϕ, degrees
- c_a = adhesion of soil to base ≤ c, ksf
- c = soil cohesion or undrained shear strength C_u, ksf
- δ = base tilt from horizontal, upward +, degrees
- β = slope of ground from base, downward +, degrees
- k = D/B if D/B ≤ 1 OR $\tan^{-1}(D/B)$ if D/B > 1 (in radians)
- D = foundation depth, ft
- Q = vertical load on foundation, kips
- T = horizontal load ≤ $Q\tan\phi + A_e c_a$, right +, kips
- A_e = effective area of foundation $B'W'$, ft^2
- m = $\frac{2 + R_{BW}}{1 + R_{BW}}$, $R_{BW} = B/W$ if T parallel to B, $R_{BW} = W/B$ if T parallel to W

f. **Computer Solutions.** Analyses by computer programs provide effective methods of estimating ultimate and allowable bearing capacities.

(1) **Program CBEAR.** Program CBEAR (Mosher and Pace 1982) can be used to calculate the bearing capacity of shallow strip, rectangular, square, or circular footings on one or two soil layers. This program uses the Meyerhof and Vesic bearing capacity factors and correction factors.

(2) **Program UTEXAS2.** UTEXAS2 is a slope stability program that can be used to calculate factors of safety for long wall footings and embankments consisting of multilayered soils (Edris 1987). Foundation loads are applied as surface pressures on flat surfaces or slopes. Circular or noncircular failure surfaces may be assumed. Noncircular failure surfaces may be straight lines and include wedges. Shear surfaces are directed to the left of applied surface loading on horizontal slopes or in the direction in which gravity would produce sliding on nonhorizontal slopes (e.g., from high toward low elevation points). This program can also consider the beneficial effect of internal reinforcement in the slope. q_u calculated by UTEXAS2 may be different from that calculated by CBEAR partly because the FS is defined in UTEXAS2 as the available shear strength divided by the shear stress on the failure surface. The assumed failure surfaces in CBEAR are not the same as the minimum FS surface found in UTEXAS2 by trial and error. FS in Table 1-2 are typical for CBEAR. Program UTEXAS2 calculates factors of safety, but these FS have not been validated with field experience. UTEXAS2 is recommended as a supplement to the Terzaghi, Meyerhof, Hansen, and Vesic models until FS determined by UTEXAS2 have been validated.

g. **Multilayer Soils.** Foundations are often supported by multilayer soils. Multiple soil layers influence the depth of the failure surface and the calculated bearing capacity. Solutions of bearing capacity for a footing in a strong layer that is overlying a weak clay layer, Figure 4-1, are given below. These solutions are valid for a punching shear failure. The use of more than two soil layers to model the subsurface soils is usually not necessary.

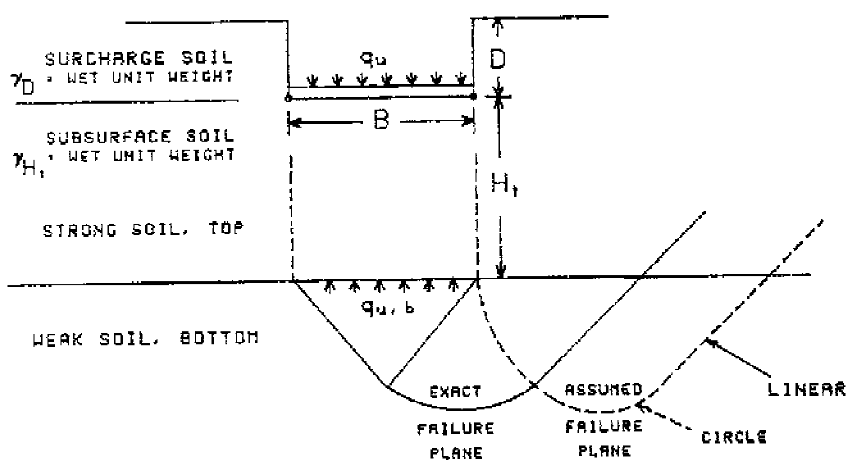


Figure 4-1. Schematic of a multilayer foundation-soil system

(1) **Depth of Analysis.** The maximum depth of the soil profile analyzed need not be much greater than the depth to the failure surface, which is approximately 2B for uniform soil. A deeper depth may be required for settlement analyses.

(a) If the soil immediately beneath the foundation is weaker than deeper soil, the critical failure surface may be at a depth less than 2B.

(b) If the soil is weaker at depths greater than 2B, then the critical failure surface may extend to depths greater than 2B.

(2) **Dense Sand Over Soft Clay.** The ultimate bearing capacity of a footing in a dense sand over soft clay can be calculated assuming a punching shear failure using a circular slip path (Hanna and Meyerhof 1980; Meyerhof 1974)

Wall Footing:

$$q_u = q_{u,b} + \frac{2\gamma_{sand}H_t^2}{B}\left(1+\frac{2D}{H_t}\right)K_{ps}\tan\phi_{sand} - \gamma_{sand}H_t \le q_{ut} \tag{4-10a}$$

Circular Footing:

$$q_u = q_{u,b} + \frac{2\gamma_{sand}H_t^2}{B}\left(1+\frac{2D}{H_t}\right)S_sK_{ps}\tan\phi_{sand} - \gamma_{sand}H_t \le q_{ut} \tag{4-10b}$$

where

$q_{u,b}$ = ultimate bearing capacity on a very thick bed of the bottom soft clay layer, ksf
γ_{sand} = wet unit weight of the upper dense sand, kips/ft^3
H_t = depth below footing base to soft clay, ft
D = depth of footing base below ground surface, ft
K_{ps} = punching shear coefficient, Figure 4-2a,4-2b, and 4-2c
ϕ_{sand} = angle of internal friction of upper dense sand, degrees
S_s = shape factor
q_{ut} = ultimate bearing capacity of upper dense sand, ksf

The punching shear coefficient k_{ps} can be found from the charts in Figure 4-2 using the undrained shear strength of the lower soft clay and a punching shear parameter C_{ps}. C_{ps}, ratio of ζ/ϕ_{sand} where ζ is the average mobilized angle of shearing resistance on the assumed failure plane, is found from Figure 4-2d using ϕ_{sand} and the bearing capacity ratio R_{bc}. $R_{bc} = 0.5\gamma_{sand}BN_\gamma/(C_uN_c)$. B is the diameter of a circular footing or width of a wall footing. The shape factor S_s, which varies from 1.1 to 1.27, may be assumed unity for conservative design.

(3) **Stiff Over Soft Clay.** Punching shear failure is assumed for stiff over soft clay.

(a) D = 0.0. The ultimate bearing capacity can be calculated for a footing on the ground surface by (Brown and Meyerhof 1969)

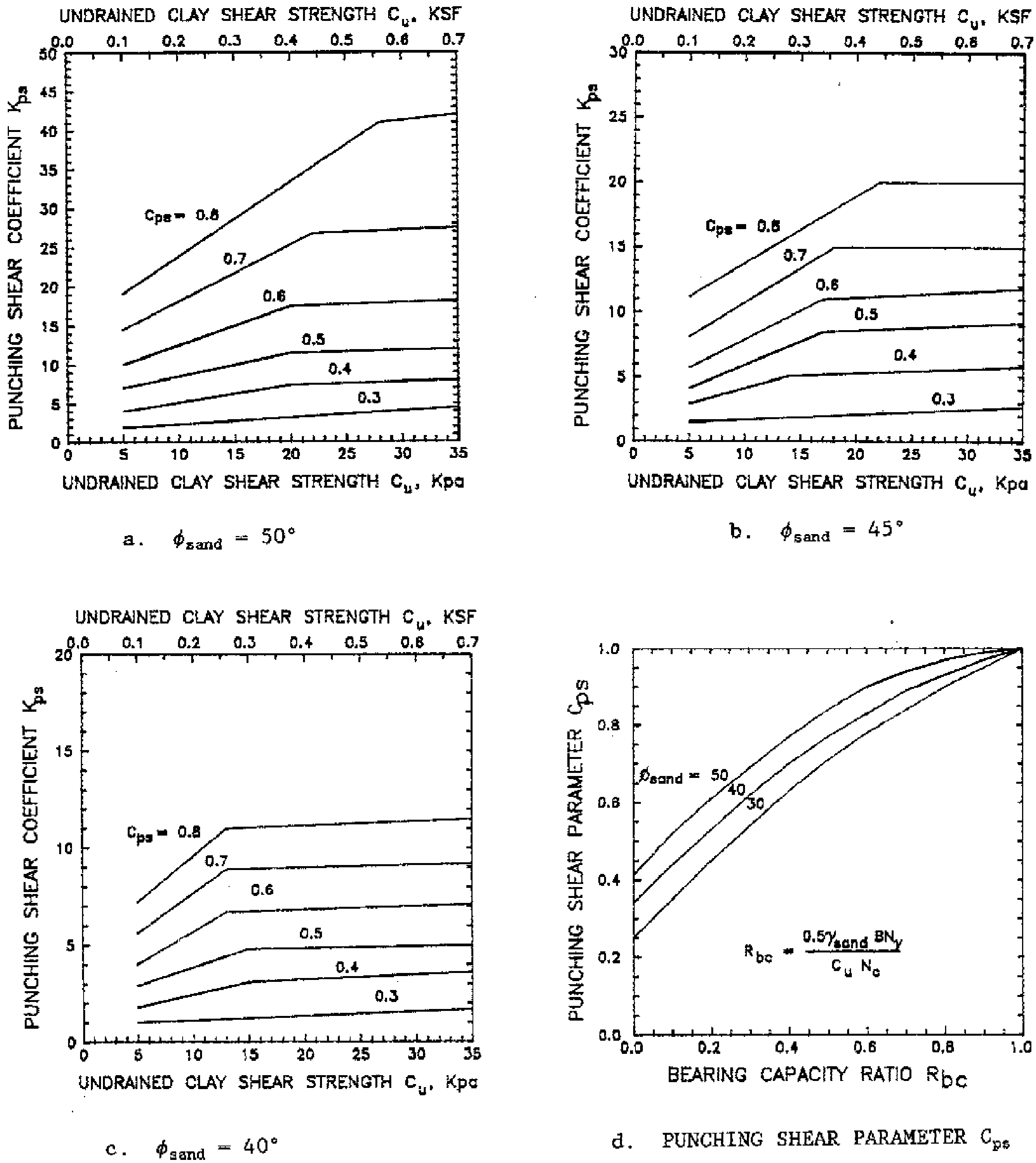


Figure 4-2. Charts for calculation of ultimate bearing capacity of dense sand over soft clay (Data from Hanna and Meyerhof 1980)

Wall Footing:

$$q_u = C_{u,upper} N_{cw,0} \tag{4-11a}$$

$$N_{cw,0} = 1.5 \frac{H_t}{B_{dia}} + 5.14 \frac{C_{u,lower}}{C_{u,upper}} \leq 5.14 \tag{4-11b}$$

Circular Footing:

$$q_u = C_{u,upper} N_{cc,0} \tag{4-11c}$$

$$N_{cc,0} = 3.0 \frac{H_t}{B_{dia}} + 6.05 \frac{C_{u,lower}}{C_{u,upper}} \leq 6.05 \tag{4-11d}$$

where

$C_{u,upper}$ = undrained shear strength of the stiff upper clay, ksf
$C_{u,lower}$ = undrained shear strength of the soft lower clay, ksf
$N_{cw,0}$ = bearing capacity factor of the wall footing
$N_{cc,0}$ = bearing capacity factor of the circular footing
B_{dia} = diameter of circular footing, ft

A rectangular footing may be converted to a circular footing by $B_{dia} = 2(BW/\pi)^{1/2}$ where B = width and W = length of the footing. Factors $N_{cw,0}$ and $N_{cc,0}$ will overestimate bearing capacity by about 10 percent if $C_{u,lower}/C_{u,upper} \geq 0.7$. Refer to Brown and Meyerhof (1969) for charts of $N_{cw,0}$ and $N_{cc,0}$.

(b) D > 0.0. The ultimate bearing capacity can be calculated for a footing placed at depth D by

Wall Footing:

$$q_u = C_{u,upper} N_{cw,D} + \gamma D \tag{4-12a}$$

Circular Footing:

$$q_u = C_{u,upper} N_{cc,D} + \gamma D \tag{4-12b}$$

where

$N_{cw,D}$ = bearing capacity factor of wall footing with D > 0.0
$N_{cc,D}$ = bearing capacity factor of rectangular footing with D > 0.0
= $N_{cw,D}[1 + 0.2(B/W)]$
γ = wet unit soil weight of upper soil, kips/ft^3
D = depth of footing, ft

$N_{cw,D}$ may be found using Table 4-7 and $N_{cw,0}$ from Equation 4-11b. Refer to Department of the Navy (1982) for charts that can be used to calculate bearing capacities in two layer soils.

(4) **Computer Analysis.** The bearing capacity of multilayer soils may be estimated from computer solutions using program CBEAR (Mosher and Pace 1982). Program UTEXAS2 (Edris 1987) calculates FS for wall footings and embankments, which have not been validated with field experience. UTEXAS2 is recommended as a supplement to CBEAR until FS have been validated.

TABLE 4-7

Influence of Footing Depth D
(Department of the Navy 1982)

D/B	$N_{cw,D}/N_{cw,0}$
0.0	1.00
0.5	1.15
1.0	1.24
2.0	1.36
3.0	1.43
4.0	1.46

h. **Correction for Large Footings and Mats.** Bearing capacity, obtained using Equation 4-1 and the bearing capacity factors, gives capacities that are too large for widths $B > 6$ ft. This is apparently because the $0.5 \cdot B'\gamma'_H N_\gamma \zeta_\gamma$ term becomes too large (DeBeer 1965; Vesic 1969).

(1) Settlement usually controls the design and loading of large dimensioned structures because the foundation soil is stressed by the applied loads to deep depths.

(2) Bearing capacity may be corrected for large footings or mats by multiplying the surcharge term $0.5 \cdot B'\gamma'_H N_\gamma \zeta_\gamma$ by a reduction factor (Bowles 1988)

$$r_\gamma = 1 - 0.25 \log_{10} \frac{B}{6} \tag{4-13}$$

where $B > 6$ ft.

i. **Presumptive Bearing Capacity.** Refer to Table 4-8 for typical presumptive allowable bearing pressures q_{na}. Presumptive allowable pressures should only be used with caution for spread footings supporting small or temporary structures and verified, if practical, by performance of nearby structures. Further details are given in Chapter 4 of Department of the Navy (1982).

(1) Bearing pressures produced by eccentric loads that include dead plus normal live loads plus permanent lateral loads should not exceed q_{na} pressures of Table 4-8.

(2) Transient live loads from wind and earthquakes may exceed the allowable bearing pressure by up to one-third.

(3) For footings of width $B < 3$ ft in least lateral dimension the allowable bearing pressures is B times 1/3 of q_{na} given in Table 4-8.

TABLE 4-8

Presumptive Allowable Bearing Pressures for Spread Footings
(Data from Department of the Navy 1982, Table 1, Chapter 4)

Bearing Material	In Place Consistency	Nominal Allowable Bearing Pressure q_{na}, ksf
Massive crystalline igneous and metamorphic rock: granite, diorite, basalt, gneiss, thoroughly cemented conglomerate (sound condition allows minor cracks)	Hard sound rock	160
Foliated metamorphic rock: slate, schist (sound condition allows minor cracks)	Medium hard sound rock	70
Sedimentary rock; hard cemented shales, siltstone, sandstone, limestone without cavities	Medium hard sound rock	40
Weathered or broken bed rock of any kind except highly argillaceousrock (shale); Rock Quality Designation less than 25	Soft rock	20
Compaction shale or other highly argillaceous rock in sound condition	Soft rock	20
Well-graded mixture of fine and coarse-grained soil: glacial till, hardpan, boulder clay (GW-GC, GC, SC)	Very compact	20
Gravel, gravel-sand mixtures, boulder gravel mixtures (SW, SP, SW, SP)	Very compact	14
	Medium to compact	10
	Loose	6
Coarse to medium sand, sand with little gravel (SW, SP)	Very compact	8
	Medium to compact	6
	Loose	3
Fine to medium sand, silty or clayey medium to coarse sand (SW, SM, SC)	Very compact	6
	Medium to compact	5
	Loose	3
Homogeneous inorganic clay, sandy or silty clay (CL, CH)	Very stiff to hard	8
	Medium to stiff	4
	Soft	1
Inorganic silt, sandy or clayey silt, varved silt-clay-fine sand	Very stiff to hard	6
	Medium to stiff	3
	Soft	1

(4) For a bearing stratum underlain by weaker material, pressure on the weak stratum should be less than the nominal allowable bearing pressure given in Table 4-8

$$\frac{Q}{(B+1.16H_t)\cdot(W+1.16H_t)} \le q_{na} \tag{4-14}$$

where

- Q = vertical load on foundation, kips
- B = foundation width, ft
- W = foundation lateral length, ft
- H_t = depth to weak stratum beneath bottom of foundation, ft
- q_{na} = nominal allowable bearing pressure, ksf

(5) Resistance to uplift force Q_{up} should be

$$\frac{W'_T}{Q_{up}} > 2 \tag{4-15}$$

where W'_T is the total effective weight of soil and foundation resisting uplift.

4-3. **Retaining Walls.**

a. **Ultimate Bearing Capacity.** Ultimate bearing capacity of retaining walls may be estimated by Equation 4-1 with dimensionless factors provided by the Meyerhof, Hansen, or Vesic methods described in Tables 4-3, 4-5, and 4-6, respectively. The dimensionless correction factors need consider only depth and load inclination for retaining walls. Equation 4-1 may be rewritten

$$q_u = cN_c\zeta_{cd}\zeta_{ci} + \frac{1}{2}B'\gamma'_H N_\gamma\zeta_{\gamma d}\zeta_{\gamma i} + \sigma'_D N_q\zeta_{qd}\zeta_{qi} \tag{4-16}$$

where N_c, N_γ, N_q and $\zeta_c, \zeta_\gamma, \zeta_q$ are given in Tables 4-3, 4-4, 4-5, or 4-6. If Hansen's model is used, then the exponent for $\zeta_{\gamma i}$ and ζ_{qi} in Table 4-5 should be changed from 5 to 2 (Bowles 1988).

b. **Allowable Bearing Capacity.** The allowable bearing capacity may be estimated from Equations 1-2 using FS = 2 for cohesionless soils and FS = 3 for cohesive soils.

4-4. **In Situ Modeling of Bearing Pressures.**

In situ load tests of the full size foundation are not usually done, except for load testing of piles and drilled shafts. Full scale testing is usually not performed because required loads are usually large and as a result these tests are expensive. The most common method is to estimate the bearing capacity of the soil from the results of relatively simple, less expensive in situ tests such as plate bearing, standard penetration, cone penetration, and vane shear tests.

a. **Plate Bearing Test.** Loading small plates 12 to 30 inches in diameter or width B_p are quite useful, particularly in sands, for estimating the bearing capacity of foundations. The soil strata within a depth 4B beneath the foundation must be similar to the strata beneath the plate. Details of this test are described in standard method ASTM D 1194. A large vehicle can be used to provide reaction for the applied pressures.

(1) **Constant Strength.** The ultimate bearing capacity of the foundation in cohesive soil of constant shear strength may be estimated by

$$B < 4B_p: \quad q_u = q_{u,p} \tag{4-17a}$$

where

q_u = ultimate bearing capacity of the foundation, ksf
$q_{u,p}$ = ultimate bearing capacity of the plate, ksf
B = diameter or width of the foundation, ft
B_p = diameter or width of the plate, ft

(2) **Strength Increasing Linearly With Depth.** The ultimate bearing capacity of the foundation in cohesionless or cohesive soil with strength increasing linearly with depth may be estimated by

$$B < 4B_p: \quad q_u = q_{u,p}\frac{B}{B_p} \tag{4-17b}$$

(3) **Extrapolation of Settlement Test Results in Sands.** The soil pressure q_1 may be estimated using a modified Terzaghi and Peck approximation (Peck and Bazarra 1969; Peck, Hanson, and Thornburn 1974)

$$q_1 = \frac{q}{1.5 \cdot \rho_i} \tag{4-18}$$

where

q_1 = soil pressure per inch of settlement, ksf/in.
q = average pressure applied on plate, ksf
ρ_i = immediate settlement of plate, in.

The results of the plate load test should indicate that q/ρ_i is essentially constant. q_1 and plate diameter B_p can then be input into the Terzaghi and Peck chart for the appropriate D/B ratio, which is 1, 0.5 or 0.25 (see Figure 3-3, EM 1110-1-1904). The actual footing dimension B is subsequently input into the same chart to indicate the allowable foundation bearing pressure.

(4) **Extrapolation of Test Results.** Load tests performed using several plate sizes may allow extrapolation of test results to foundations up to 6 times the plate diameter provided the soil is similar. Other in situ test results using standard penetration or cone penetration data are recommended for large foundation diameters and depths more than $4B_p$.

b. **Standard Penetration Test (SPT).** The SPT may be used to directly obtain the allowable bearing capacity of soils for specific amounts of settlement based on past correlations.

(1) **Footings.** Meyerhof's equations (Meyerhof 1956; Meyerhof 1974) are modified to increase bearing capacity by 50 percent (Bowles 1988)

$$B \le 4\ ft: \qquad q_{a,1} = \frac{N_n}{F_1} K_d \tag{4-19a}$$

$$B > 4\ ft: \qquad q_{a,1} = \frac{N_n}{F_2}\left[\frac{B+F_3}{B}\right]^2 \tag{4-19b}$$

where

$q_{a,1}$ = allowable bearing capacity for 1 inch of settlement, ksf
K_d = $1 + 0.33(D/B) \le 1.33$
N_n = standard penetration resistance corrected to n percent energy

Equation 4-19b may be used for footings up to 15 ft wide.

(a) F factors depend on the energy of the blows. n is approximately 55 percent for uncorrected penetration resistance and $F_1 = 2.5$, $F_2 = 4$, and $F_3 = 1$. F factors corrected to $n = 70$ percent energy are $F_1 = 2$, $F_2 = 3.2$ and $F_3 = 1$.

(b) Figure 3-3 of EM 1110-1-1904 provides charts for estimating q_a for 1 inch of settlement from SPT data using modified Terzaghi and Peck approximations.

(2) **Mats.** For mat foundations

$$q_{a,1} = \frac{N_n}{F_2} K_d \tag{4-20a}$$

where $q_{a,1}$ is the allowable bearing capacity for limiting settlement to 1 inch. The allowable bearing capacity for any settlement q_a may be linearly related to the allowable settlement for 1 inch obtained from Equations 4-19 assuming settlement varies in proportion to pressure

$$q_a = \rho \cdot q_{a,1} \tag{4-20b}$$

where

ρ = settlement, inches
$q_{a,1}$ = allowable bearing capacity for 1 inch settlement, ksf

c. **Cone Penetration Test (CPT).** Bearing capacity has been correlated with cone tip resistance q_c for shallow foundations with $D/B \leq 1.5$ (Schmertmann 1978).

(1) The ultimate bearing capacity of cohesionless soils is given by

$$\textit{Strip}: \quad q_u = 28 - 0.0052(300 - q_c)^{1.5} \tag{4-21a}$$

$$\textit{Square}: \quad q_u = 48 - 0.0090(300 - q_c)^{1.5} \tag{4-21b}$$

where q_u and q_c are in units of tsf or kg/cm^2.

(2) The ultimate bearing capacity of cohesive soils is

$$\textit{Strip}: \quad q_u = 2 + 0.28 q_c \tag{4-22a}$$

$$\textit{Square}: \quad q_u = 5 + 0.34 q_c \tag{4-22b}$$

Units are also in tsf or kg/cm^2. Table 4-9 using Figure 4-3 provides a procedure for estimating q_u for footings up to B = 8 ft in width.

d. **Vane Shear Test.** The vane shear is suitable for cohesive soil because bearing capacity is governed by short-term, undrained loading which is simulated by this test. Bearing capacity can be estimated by (Canadian Geotechnical Society 1985)

$$q_u = 5 \cdot R_v \cdot \tau_u \left[1 + 0.2 \cdot \frac{D}{B}\right]\left[1 + 0.2 \cdot \frac{B}{L}\right] + \sigma_{vo} \tag{4-24}$$

where

R_v = strength reduction factor, Figure 4-4
τ_u = field vane undrained shear strength measured during the test, ksf
D = depth of foundation, ft
B = width of foundation, ft
L = length of foundation, ft
σ_{vo} = total vertical soil overburden pressure at the foundation level, ksf

4-5. **Examples.** Estimation of the bearing capacity is given below for (1) a wall footing placed on the ground surface subjected to a vertical load, (2) a rectangular footing placed below the ground surface and subjected to an inclined load, and (3) a tilted, rectangular footing on a slope and subjected to an eccentric load. Additional examples are provided in the user manual for CBEAR (Mosher and Pace 1982). The slope stability analysis of embankments is described in the user manual for UTEXAS2 (Edris 1987). Bearing capacity analyses should be performed using at least three methods where practical.

TABLE 4-9

CPT Procedure for Estimating Bearing Capacity
of Footings on Cohesive Soil (Data from Tand, Funegard, and Briaud 1986)

Step	Procedure
1	Determine equivalent $\bar{q}_c$ from footing base to 1.5B below base by $$\overline{q_c} = (q_{cb1} \cdot q_{cb2})^{0.5} \qquad (4\text{-}23a)$$ where $\bar{q}_c$ = equivalent cone tip bearing pressure below footing, ksf q_{cb1} = average tip resistance from 0.0 to 0.5B, ksf q_{cb2} = average cone tip resistance from 0.5B to 1.5B, ksf
2	Determine equivalent depth of embedment D_e , ft, to account for effect of strong or weak soil above the bearing elevation $$D_e = \sum_{i=1}^{n} \Delta z_i \frac{q_{ci}}{\overline{q_c}} \qquad (4\text{-}23b)$$ where n = number of depth increments to depth D D = unadjusted (actual) depth of embedment, ft Δz_i = depth increment i, ft q_{ci} = cone tip resistance of depth increment i, ksf q_c = equivalent cone tip bearing pressure below footing, ksf
3	Determine ratio of equivalent embedment depth to footing width $$R_d = \frac{D_e}{B} \qquad (4\text{-}23c)$$
4	Estimate bearing ratio R_k from Figure 4-3 using R_d. The lower bound curve is applicable to fissured or slickensided clays. The average curve is applicable to all other clays unless load tests verify the upper bound curve for intact clay.
5	Estimate total overburden pressure σ_{vo}, then calculate $$q_{ua} = R_k(\overline{q_c} - \sigma_{vo}) + \sigma_{vo} \qquad (4\text{-}23d)$$ where q_{ua} = ultimate bearing capacity of axially loaded square or round footings with horizontal ground surface and base. Adjust q_{ua} obtained from Equation 4-23d for shape, eccentric loads, sloping ground or tilted base using Hansen's factors for cohesion, Table 4-5, to obtain the ultimate capacity $$q_u = \zeta_c \cdot q_{ua} \qquad (4\text{-}23e)$$ where ζ_c is defined by Equation 4-8a.

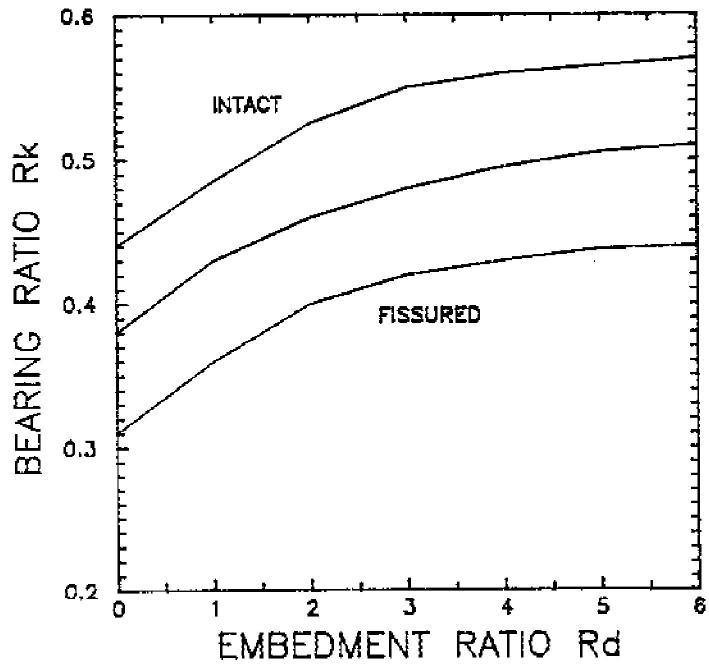


Figure 4-3. Bearing ratio R_k for axially loaded square and round footings (Data from Tand, Funegard, and Briaud 1986)

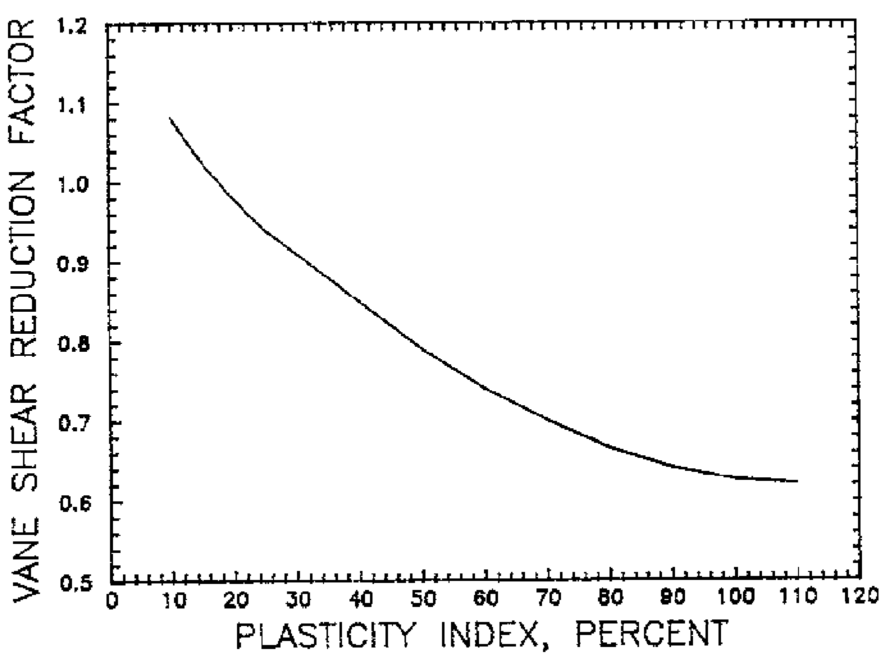


Figure 4-4. Strength reduction factor for field vane shear (Data from Bjerrum 1973)

a. **Wall Footing.** A wall footing 3 ft wide with a load Q = 12 kips/ft (bearing pressure q = 4 ksf) is proposed to support a portion of a structure in a selected construction site. The footing is assumed to be placed on or near the ground surface for this analysis such that D = 0.0 ft, Figure 4-5, and $\sigma'_D = 0.0$. Depth H is expected to be < 2B or < 6 ft.

(1) **Soil Exploration.** Soil exploration indicated a laterally uniform cohesive soil in the proposed site. Undrained triaxial compression test results were performed on specimens of undisturbed soil samples to determine the undrained shear strength. Confining pressures on these specimens were equal to the total vertical overburden pressure applied to these specimens when in the field. Results of these tests indicated the distribution of shear strength with depth shown in Figure 4-6. The minimum shear strength $c = C_u$ of 1.4 ksf observed 5 to 7 ft below ground surface is selected for the analysis. The friction angle is $\phi = 0.0$ deg and the wet unit weight is 120 psf.

(2) **Ultimate Bearing Capacity**

(a) Terzaghi Method. Table 4-1 indicates $N_c = 5.7$, $N_q = 1.0$ and $N_\gamma = 0.00$. The total ultimate capacity q_u is

$$q_U = cN_C = 1.4 \cdot 5.7 = 8.0 \; ksf$$

The Terzaghi method indicates an ultimate bearing capacity $q_u = 8$ ksf.

(b) Meyerhof Method. The ultimate bearing capacity of this wall footing using program CBEAR yields $q_u = 7.196$ ksf. The Hansen and Vesic solutions will be similar.

(3) **Allowable Bearing Capacity.** FS for this problem from Table 1-2 is 3.0. Therefore, q_a using Equation 1-2a is $q_u/FS = 8.000/3 = 2.7$ ksf from the Terzaghi solution and 7.196/3 = 2.4 ksf from CBEAR. The solution using program UTEXAS2 gives a minimum FS = 2.2 for a circular failure surface of radius 3 ft with its center at the left edge of the footing.

(4) **Recommendation.** q_a ranges from 2.4 to 2.7 while the proposed design pressure q_d is 4 ksf. q_d should be reduced to 2.4 ksf $\leq q_a$.

b. **Rectangular Footing With Inclined Load.** A rectangular footing with B = 3 ft, W = 6 ft, D = 2 ft, similar to Figure 1-6, is to be placed in cohesionless soil on a horizontal surface ($\beta = 0.0$) and without base tilt ($\delta = 0.0$). The effective friction angle $\phi' = 30$ deg and cohesion $c = c_u = 0.0$. The surcharge soil has a wet (moist) unit weight $\gamma_D = 0.120$ kip/ft^3 (120 pcf), subsurface soil has a wet (moist) unit weight $\gamma_H = 0.130$ kip/ft^3 (130 pcf), and depth to groundwater is D_{GWT} = 3 ft. The saturated unit weight is assumed the same as the wet unit weight. The applied vertical load on the foundation is Q = 10 kips and the horizontal load T = +2 kips to the right.

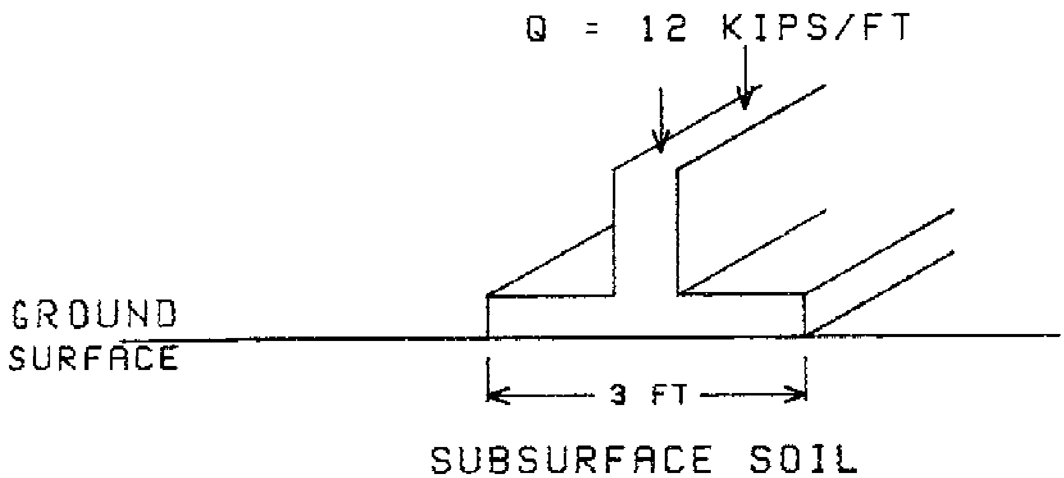


Figure 4-5. Example wall footing bearing capacity analysis

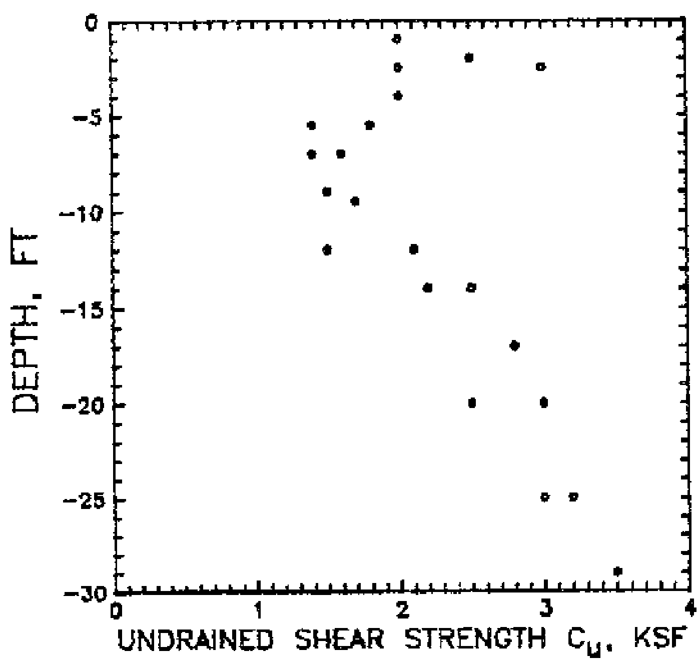


Figure 4-6. Example undrained shear strength distribution with depth

(1) **Effective Stress Adjustment.** Adjust the unit soil weights due to the water table using Equation 1-6

$$\gamma_{HSUB} = \gamma_H - \gamma_w = 0.130 - 0.0625 = 0.0675 \text{ kip/ft}^3$$
$$H = B \cdot \tan(45 + \phi/2) = 3.00 \cdot 1.73 = 5.2 \text{ ft}$$
$$\gamma'_H = \gamma_{HSUB} + [(D_{GWT} - D)/H] \cdot \gamma_w = 0.0675 + [(3 - 2)/5.2] \cdot 0.0625$$
$$= 0.08 \text{ kip/ft}^3$$

From Equation 1-7a, $\sigma'_D = \sigma_D = \gamma_D \cdot D = 0.120 \cdot 2.00 = 0.24$ ksf

(2) **Meyerhof Method.** For $\phi' = 30$ deg, $N_q = 18.40$, $N_\gamma = 15.67$, and $N_\phi = 3.00$ from Table 4-4. N_c is not needed since $c = 0.0$. From Table 4-3,

(a) Wedge correction factor $\zeta_\gamma = \zeta_{\gamma s} \cdot \zeta_{\gamma i} \cdot \zeta_{\gamma d}$

$$\zeta_{\gamma s} = 1 + 0.1 \cdot N_\phi \cdot (B'/W') = 1 + 0.1 \cdot 3.00 \cdot (3/6) = 1.15$$
$$R = (Q^2 + T^2)^{0.5} = (100 + 4)^{0.5} = 10.2$$
$$\theta = \cos^{-1}(Q/R) = \cos^{-1}(10/10.2) = 11.4 \text{ deg} < \phi = 30 \text{ deg}$$
$$\zeta_{\gamma i} = [1 - (\theta/\phi')]^2 = [1 - (11.4/30)]^2 = 0.384$$
$$\zeta_{\gamma d} = 1 + 0.1\sqrt{N_\phi} \cdot (D/B) = 1 + 0.1 \cdot 1.73 \cdot (2/3) = 1.115$$
$$\zeta = 1.15 \cdot 0.384 \cdot 1.115 = 0.49$$

(b) Surcharge correction factor $\zeta_q = \zeta_{qs} \cdot \zeta_{qi} \cdot \zeta_{qd}$

$$\zeta_{qs} = \zeta_{\gamma s} = 1.15$$
$$\zeta_{qi} = [1 - (\theta/90)]^2 = [1 - (11.4/90)]^2 = 0.763$$
$$\zeta_{qd} = \zeta_{\gamma d} = 1.115$$
$$\zeta_q = 1.15 \cdot 0.763 \cdot 1.115 = 0.98$$

(c) Total ultimate bearing capacity from Equation 4-1 is

$$q_u = 0.5 \cdot B \cdot \gamma'_H N_\gamma \cdot \zeta_\gamma + \sigma'_D \cdot N_q \cdot \zeta_q$$
$$= 0.5 \cdot 3.00 \cdot 0.08 \cdot 15.67 \cdot 0.49 + 0.24 \cdot 18.40 \cdot 0.98$$
$$= 0.92 + 4.33 = 5.25 \text{ ksf}$$

(3) **Hansen Method.** For $\phi' = 30$ deg, $N_q = 18.40$, $N_\gamma = 15.07$, and $N_\phi = 3.00$ from Table 4-4. N_c is not needed since $c = 0.0$. From Table 4-5,

(a) Wedge correction factor $\zeta_\gamma = \zeta_{\gamma s} \cdot \zeta_{\gamma i} \cdot \zeta_{\gamma d} \cdot \zeta_{\gamma \beta} \cdot \zeta_{\gamma \delta}$ where $\zeta_{\gamma \beta} = \zeta_{\gamma \delta} = 1.00$

$$\zeta_{\gamma s} = 1 - 0.4 \cdot (B'/W') = 1 - 0.4 \cdot (3/6) = 0.80$$
$$\zeta_{\gamma i} = [1 - (0.7T/Q)]^5 = [1 - (0.7 \cdot T/10)]^5 = 0.47$$
$$\zeta_{\gamma d} = 1.00$$
$$\zeta_\gamma = 0.80 \cdot 0.47 \cdot 1.00 = 0.376$$

(b) Surcharge correction factor $\zeta_q = \zeta_{qs} \cdot \zeta_{qi} \cdot \zeta_{qd} \cdot \zeta_{q\beta} \cdot \zeta_{q\delta}$ where $\zeta_{q\beta} = \zeta_{q\delta} = 1.00$

$\zeta_{qs} = 1 + (B/W)\cdot \tan\phi = 1 + (3/6)\cdot 0.577 = 1.289$
$\zeta_{qi} = [1 - (0.5T/Q)]^5 = [1 - (0.5\cdot 2/10)]^5 = 0.59$
$k = D/B = 2/3$
$\zeta_{qd} = 1 + 2\cdot \tan\phi'\cdot(1 - \sin\phi')^2\cdot k = 1 + 2\cdot 0.577\cdot(1 - 0.5)^2 \cdot 2/3$
$= 1.192$
$\zeta_q = 1.289\cdot 0.59\cdot 1.192 = 0.907$

(c) Total ultimate bearing capacity from Equation 4-1 is

$$
\begin{aligned}
q_u &= 0.5\cdot B\cdot \gamma'_H\cdot N_\gamma\cdot \zeta_\gamma + \sigma'_D\cdot N_q\cdot \zeta_q \\
&= 0.5\cdot 3.00\cdot 0.08\cdot 15.07\cdot 0.376 + 0.24\cdot 18.40\cdot 0.907 \\
&= 0.68 + 4.01 = 4.69 \text{ ksf}
\end{aligned}
$$

(4) **Vesic Method.** For $\phi' = 30$ deg, $N_q = 18.40$, $N_\gamma = 22.40$, and $N_\phi = 3.00$ from Table 4-4. N_c is not needed. From Table 4-6,

(a) Wedge correction factor $\zeta_\gamma = \zeta_{\gamma s}\cdot \zeta_{\gamma i}\cdot \zeta_{\gamma d}\cdot \zeta_{\gamma\beta}\cdot \zeta_{\gamma\delta}$ where $\zeta_{\gamma\beta} = \zeta_{\gamma\delta} = 1.00$

$\zeta_{\gamma s} = 1 - 0.4\cdot B/W = 1 - 0.4\cdot 3/6 = 0.80$
$R_{BW} = B/W = 3/6 = 0.5$
$m = (2 + R_{BW})/(1 + R_{BW}) = (2 + 0.5)/(1 + 0.5) = 1.67$

$\zeta_{\gamma i} = [(1 - (T/Q)]^{m+1} = [1 - (2/10)]^{1.67+1} = 0.551$

$\zeta_\gamma = 0.80\cdot 0.551\cdot 1.00 = 0.441$

(b) Surcharge correction factor $\zeta_q = \zeta_{qs}\cdot \zeta_{qi}\cdot \zeta_{qd}\cdot \zeta_{q\beta}\cdot \zeta_{q\delta}$ where $\zeta_{q\beta} = \zeta_{q\delta} = 1.00$

$\zeta_{qs} = 1 + (B/W)\cdot \tan\phi = 1 + 3/6\cdot 0.577 = 1.289$
$\zeta_{qi} = [1 - (T/Q)]^m = [1 - (2/10)]^m = 0.689$
$\zeta_{qd} = 1 + 2\cdot \tan\phi'\cdot(1 - \sin\phi')^2\cdot k = 1 + 2\cdot 0.577\cdot(1 - 0.5)\cdot 2/3$
$= 1.192$
$\zeta_q = 1.289\cdot 0.689\cdot 1.192 = 1.058$

(c) Total ultimate bearing capacity from Equation 3-1a is

$$
\begin{aligned}
q_u &= 0.5\cdot B\cdot \gamma'_H\cdot N_\gamma\cdot \zeta_\gamma + \sigma'_D\cdot N_q\cdot \zeta_q \\
&= 0.5\cdot 3.00\cdot 0.08\cdot 22.40\cdot 0.441 + 0.24\cdot 18.40\cdot 1.058 \\
&= 1.19 + 4.67 = 5.86 \text{ ksf}
\end{aligned}
$$

(5) **Program CBEAR.** Zero elevation for this problem is defined 3 ft below the foundation base. Input to this program is as follows (refer to Figure 1-6):

(a) Foundation coordinates: $x_1 = 10.00$, $y_1 = 3.00$
$x_2 = 13.00$, $y_2 = 3.00$
Length of footing: = 6.00

(b) Soil Coordinates: (top elevation of subsurface soil) $x_1 = x_{s1} = 10.00$, $y_1 = y_{s1} = 3.00$
$x_2 = x_{s2} = 13.00$, $y_2 = y_{s2} = 3.00$

(c) Soil Properties: (subsurface soil)

moist (wet) unit weight γ_H	= 130 pounds/ft^3
saturated unit weight	= γ_H
friction angle	= 30 deg
cohesion	= 0.00

(d) Options:

<u>One surcharge layer</u>

y coordinate of top of surcharge	= 5.00 ft
moist unit weight	= 120 pounds/ft^3
saturated unit weight	= 120 pounds/ft^3

<u>Water table description</u>

y coordinate of top of water table	= 2.00 ft
unit weight of water	= 62.5 pounds/ft^3

<u>Applied load description</u>

applied load (R)	= 10.2 kips
x coordinate of base application point	= 11.5 ft
z coordinate of base application point	= 3.00 ft
inclination of load clockwise from vertical	= 11.4 deg

(e) CBEAR calculates q_u = 5.34 ksf

(f) Comparison of methods indicates bearing capacities

Method	Total q_u, ksf	Net q_u', ksf
Meyerhof	5.25	5.01
Hansen	4.69	4.45
Vesic	5.86	5.62
Program CBEAR	5.34	5.10

The net bearing capacity is found by subtracting $\gamma_D \cdot D = 0.12 \cdot 2 = 0.24$ ksf from q_u, Equation 4-2. The resultant applied pressure on the footing is $q_r = R/(BW) = 10.2/(3 \cdot 6) = 0.57$ ksf. The factor of safety of all of the above methods with respect to the net bearing capacity is on the order of $q_u'/q_r \approx 9$. The Hansen method is most conservative.

c. **Rectangular Footing With Eccentricity, Base Tilt, and Ground Slope.** A rectangular footing, B = 3 ft and W = 5 ft, is placed in a cohesionless soil with base tilt δ = 5 deg and ground slope β = 15 deg as illustrated in Table 4-5 and Figure 4-7. ϕ' = 26 deg and c = c_a = 0.0. Soil wet unit weight γ_D = 120 lbs/ft^3, subsurface soil wet unit weight γ_H = 130 lbs/ft^3, and depth to groundwater D_{GWT} = 3 ft. Vertical applied load Q = 10 kips and horizontal load T = 0 kips, but M_B = 5 kips-ft and M_W = 10 kips-ft.

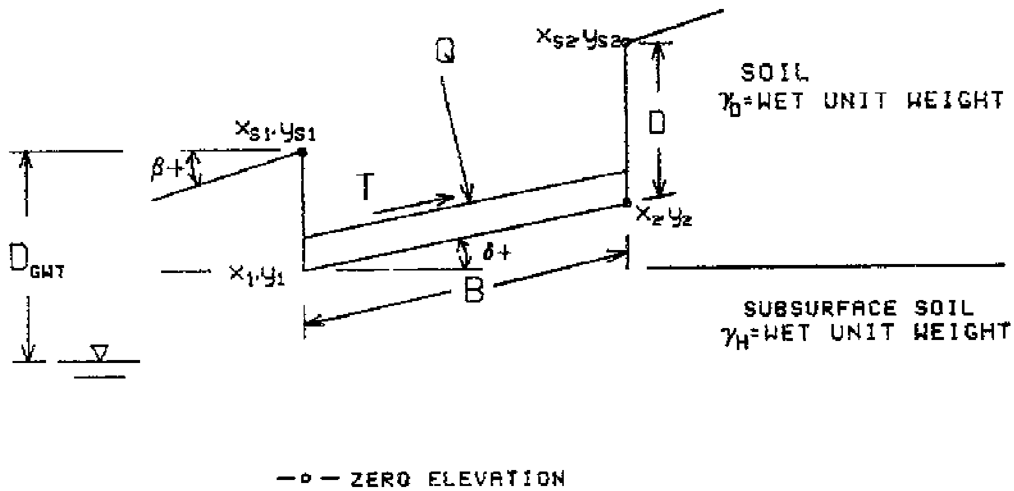


Figure 4-7. Shallow foundation with slope and base tilt

(1) **Coordinate Adjustment.** δ = 5 deg indicates right side elevation of the base is $3 \cdot \sin 5$ deg = 0.26 ft higher than the left side. β = 15 deg indicates right side foundation elevation at the ground surface is $3 \cdot \sin 15$ deg = 0.78 ft higher than the left side.

(2) **Effective Stress Adjustment.** Average D_{GWT} = 3 + 0.78/2 = 3.39 ft. Average D = 2 + 0.78/2 - 0.26/2 = 2.26 ft. Adjustment of soil unit wet weight for the water table from Equation 1-6 is

$$\gamma_{HSUB} = \gamma_H - \gamma_w = 0.130 - 0.0625 = 0.0675 \text{ kip/ft}^3$$
$$H = B \cdot \tan[45 + (\phi/2)] = 3.00 \cdot 1.73 = 5.2 \text{ ft}$$
$$\gamma'_H = \gamma_{HSUB} + [(D_{GWT} - D)/H] \cdot \gamma_w$$
$$= 0.0675 + [(3.39 - 2.26)/5.2] \cdot 0.0625 = 0.081 \text{ kip/ft}^3$$
$$\sigma'_D = \sigma_D = \gamma_D \cdot D = 0.120 \cdot 2.26 = 0.27 \text{ ksf}$$

(3) **Eccentricity Adjustment.** Bending moments lead to eccentricities from Equations 4-4c and 4-4d

$$e_B = M_B/Q = 5/10 = 0.5 \text{ ft}$$
$$e_W = M_W/Q = 10/10 = 1.0 \text{ ft}$$

Effective dimensions from Equations 4-4a and 4-4b are

$$B' = B - 2e_B = 3 - 2 \cdot 0.5 = 2 \text{ ft}$$
$$W' = W - 2e_W = 5 - 2 \cdot 1.0 = 3 \text{ ft}$$

(4) **Hansen Method.** For $\phi' = 26$ deg, $N_q = 11.85$ and $N_\gamma = 7.94$ from Table 4-4. N_c is not needed since c = 0.0. From Table 4-5,

(a) Wedge correction factor $\zeta_\gamma = \zeta_{\gamma s} \cdot \zeta_{\gamma i} \cdot \zeta_{\gamma d} \cdot \zeta_{\gamma\beta} \cdot \zeta_{\gamma\delta}$ where $\zeta_{\gamma i} = 1.00$

$$\zeta_{\gamma s} = 1 - 0.4 \cdot B'/W' = 1 - 0.4 \cdot 2/3 = 0.733$$
$$\zeta_{\gamma d} = 1.00$$
$$\zeta_{\gamma\beta} = (1 - 0.5 \cdot \tan\beta)^5 = (1 - 0.5 \cdot \tan 15)^5 = 0.487$$
$$\zeta_{\gamma\delta} = e^{-0.047 \cdot \delta \cdot \tan\phi'} = e^{-0.047 \cdot 5 \cdot \tan 26} = 0.892$$
$$\zeta_\gamma = 0.733 \cdot 1.000 \cdot 0.487 \cdot 0.892 = 0.318$$

(b) Surcharge correction factor $\zeta_q = \zeta_{qs} \cdot \zeta_{qi} \cdot \zeta_{qd} \cdot \zeta_{q\beta} \cdot \zeta_{q\delta}$ where $\zeta_{qi} = 1.00$

$$\zeta_{qs} = 1 + (B'/W') \cdot \tan\phi = 1 + (2/3) \cdot 0.488 = 1.325$$
$$k = D/B = 2.26/3 = 0.753$$
$$\zeta_{qd} = 1 + 2 \cdot \tan\phi' \cdot (1 - \sin\phi')^2 \cdot k = 1 + 2 \cdot 0.488 \cdot (1 - 0.438) \cdot 0.753$$
$$\zeta_{qd} = 1.232$$
$$\zeta_{q\beta} = \zeta_{\gamma\beta} = 0.487$$
$$\zeta_{q\delta} = e^{-0.035 \cdot \delta \cdot \tan\phi'} = e^{-0.035 \cdot 5 \cdot \tan 26} = 0.918$$
$$\zeta_q = 1.325 \cdot 1.232 \cdot 0.487 \cdot 0.918 = 0.730$$

(c) Total ultimate bearing capacity from Equation 4-1 is

$$\begin{aligned} q_u &= 0.5 \cdot B \cdot \gamma'_H \cdot N_\gamma \cdot \zeta_\gamma + \sigma'_D \cdot N_q \cdot \zeta_q \\ &= 0.5 \cdot 2.00 \cdot 0.081 \cdot 7.942 \cdot 0.318 + 0.27 \cdot 11.85 \cdot 0.730 \\ &= 0.205 + 2.335 = 2.54 \text{ ksf} \end{aligned}$$

(5) **Vesic Method.** For $\phi' = 26$ deg, $N_q = 11.85$ and $N_\gamma = 12.54$ from Table 4-4. N_c is not needed. From Table 4-6,

(a) Wedge correction factor $\zeta_\gamma = \zeta_{\gamma s} \cdot \zeta_{\gamma i} \cdot \zeta_{\gamma d} \cdot \zeta_{\gamma\beta} \cdot \zeta_{\gamma\delta}$ where $\zeta_{\gamma i} = 1.00$ = 1.00

$$\zeta_{\gamma s} = 1 - 0.4 \cdot B/W = 1 - 0.4 \cdot 2/3 = 0.733$$
$$\zeta_{\gamma d} = 1.00$$
$$\zeta_{\gamma\beta} = (1 - \tan\beta)^2 = (1 - \tan 15)^2 = 0.536$$
$$\zeta_{\gamma\delta} = (1 - 0.017 \cdot \delta \cdot \tan\phi')^2 = (1 - 0.017 \cdot 5 \cdot \tan 26)^2 = 0.919$$
$$\zeta_\gamma = 0.733 \cdot 1.00 \cdot 0.536 \cdot 0.919 = 0.361$$

(b) Surcharge correction factor $\zeta_q = \zeta_{qs} \cdot \zeta_{qi} \cdot \zeta_{qd} \cdot \zeta_{q\beta} \cdot \zeta_{q\delta}$ where $\zeta_{qi} = \zeta_{q\delta}$ = 1.00

$$\zeta_{qs} = 1 + (B/W) \cdot \tan\phi = 1 + 2/3 \cdot 0.488 = 1.325$$
$$\zeta_{qd} = 1 + 2 \cdot \tan\phi' \cdot (1 - \sin\phi')^2 \cdot k$$
$$\zeta_{qd} = 1 + 2 \cdot 0.488 \cdot (1 - 0.438)^2 \cdot 0.753 = 1.232$$
$$\zeta_{q\beta} = \zeta_{\gamma\beta} = 0.536$$
$$\zeta_{q\delta} = \zeta_{\gamma\delta} = 0.919$$
$$\zeta_q = 1.325 \cdot 1.232 \cdot 0.536 \cdot 0.919 = 0.804$$

(c) Total ultimate bearing capacity from Equation 4-1 is

$$\begin{aligned} q_u &= 0.5 \cdot B \cdot \gamma'_H \cdot N_\gamma \cdot \zeta_\gamma &&+ \sigma'_D \cdot N_q \cdot \zeta_q \\ &= 0.5 \cdot 2.00 \cdot 0.081 \cdot 12.54 \cdot 0.361 &&+ 0.27 \cdot 11.85 \cdot 0.804 \\ &= 0.367 &&+ 2.572 = 2.94 \text{ ksf} \end{aligned}$$

(6) Program CBEAR. Input is as follows (refer to Figure 4-7):

(a) Foundation coordinates: $x_1 = 10.00$, $y_1 = 3.00$
$x_2 = 13.00$, $y_2 = 3.26$
Length of footing: = 5.00

(b) Soil Coordinates: $x_{s1} = 10.00$, $y_{s1} = 5.00$
$x_{s2} = 13.00$, $y_{s2} = 5.78$

(c) Soil Properties: moist (wet) unit weight γ_H = 120 pounds/ft^3
saturated unit weight = γ_H
friction angle = 26 deg
cohesion = 0.00

(d) Options:

<u>One surcharge layer</u>
y coordinate of top of subsurface soil = 3.00 ft
moist unit weight = 130 pounds/ft^3
saturated unit weight = 130 pounds/ft^3
friction angle = 26 degrees
cohesion = 0.0

<u>Water table description</u>
y coordinate of top of water table = 2.00 ft
unit weight of water = 62.5 pounds/ft^3

<u>Applied load description</u>
applied load (R) = 10.0 kips
x coordinate of base application point = 11.0 ft
z coordinate of base application point = 2.00 ft
inclination of load clockwise from vertical = 0.0 deg

(e) CBEAR calculates q_u = 2.21 ksf

(f) Comparison of methods indicates bearing capacities

Method	Total q_u, ksf	Net q'_u, ksf
Hansen	2.55	2.28
Vesic	3.94	2.67
Program CBEAR	2.21	1.94

Net bearing capacity is found by subtracting $\gamma_D \cdot D = 0.12 \cdot (2 + 2.78)/2 = 0.27$ ksf from q_u, Equation 4-2. The resultant applied pressure on the footing is $q_r = Q/(B'W') = 10/(2 \cdot 3) = 1.67$ ksf. The factors of safety of all of the above methods are $q'_u/q_r < 2$. The footing is too small for the applied load and bending moments. Program CBEAR is most conservative. CBEAR ignores subsoil data if the soil is sloping and calculates bearing capacity for the footing on the soil layer only.